pected greater things from his pregnant and ſagacious Wit. For he was ſcarce 20 Years of Age when he held theſe Correſpondencies with Mr. *Crabtrie.* And at the Age of 23. he was killed at *Marſton-Moor-Battle*, on *July* 2. 1644. fighting for King *Charles* I. His Father was *Henry Gaſcoigne* Eſq; of *Middleton*, between *Leeds* and *Wakefield.*

IV. *An Attempt towards the Improvement of the Method of approximating, in the Extraction of the Roots of Equations in Numbers. By* Brook Taylor, *Secretary to the Royal Society.*

IN *Phil. Tran.* No. 210. Dr. *Halley*, now Secretary of the Royal Society, has publiſh'd a very compendious and uſeful Method of extracting the Roots of affected Equations of the common Form, in Numbers. This Method proceeds by aſſuming the Root deſired nearly true to one or two Places in Decimals (which is done by a Geometrical Conſtruction, or by ſome other convenient way) and correcting the Aſſumption by comparing the Difference between the true Root and the aſſumed, by means of a new Equation whoſe Root is that Difference, and which he ſhews how to form from the Equation propoſed, by Subſtitution of the Value of the Root ſought, partly in known and partly in unknown Terms.

In doing this he makes uſe of a Table of Products (which he calls *Speculum Analyticum,*) by which he computes the Coefficients in the new Equation for finding the Difference mentioned. This Table, I obſerved, was formed in the ſame Manner from the Equation pro-

propos'd, as the Fluxions are, taking the Root ſought for the only flowing Quantity, its Fluxion for Unity, and after every Operation dividing the Product ſucceſſively by the Numbers 1, 2, 3, 4, *&c.* Hence I ſoon found that this Method might eaſily and naturally be drawn from *Cor.* 2. *Prop.* 7. of my *Methodus Incrementorum*, and that it was capable of a further degree of Generality; it being Applicable, not only to Equations of the common Form, (*viz.* ſuch as conſiſt of Terms wherein the Powers of the Root ſought are poſitive and integral, without any Radical Sign) but alſo to all Expreſſions in general, wherein any thing is propoſed as given which by any known Method might be computed; if *vice verſâ,* the Root were conſider'd as given: ſuch as are all Radical Expreſſions of Binomials, Trinomials, or of any other Nomial, which may be computed by the Root given, at leaſt by Logarithms, whatever be the Index of the Power of that Nomial; as likewiſe Expreſſions of Logarithms, of Arches by the Sines or Tangents, of Areas of Curves by the *Abſciſſa's* or any other Fluents, or Roots of Fluxional Equations, *&c.*

For the ſake of this great Generality, it may not be improper to ſhew how this Method is derived from the foreſaid *Corollary.* Therefore z and x being two flowing Quantities (whoſe Relation to one another may be expreſt by any Equation whatſoever) by this *Corollary,* while z by flowing uniformly becomes $z + v$, x will become $x + \frac{\dot{x}}{1.\dot{z}} v + \frac{\ddot{x}}{1.2\dot{z}^2} v + \frac{\dddot{x}}{1.2.3\dot{z}^3} v^3 +$ *&c.* or $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1 \times 2} + \frac{\dddot{x}v^3}{1.2.3} +$ *&c.* for $\dot{z}$ putting 1.

Hence if y be the Root of any Expreſſion formed of y and known Quantities, and ſuppoſed equal to nothing, and

and z be a part of y, and x be formed of z and the known Quantities, in the ſame manner as the Expreſſion made equal to nothing is formed of y; and let y be equal to $z + v$: the difference v will be found by Extracting the Root of this expreſſion $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} + \frac{\dddot{x}v^3}{1.2.3}$ + &c. $= 0$. For in this Caſe z being become $z + v = y$, x, which is now become $x + \dot{x}v + \frac{\ddot{x}v^2}{2}$ + &c. muſt become equal to nothing.

The Root v in the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} + \frac{\dddot{x}v^3}{1.2.3}$ + &c. $= 0$, is to be found upon the Suppoſition of its being very ſmall with reſpect to z, (as it muſt be, if z be taken tolerably exact) by which means the Terms $\frac{\dddot{x}v^3}{1.2.3} + \frac{\ddddot{x}v^4}{1.2.3.4}$ + &c. may be neglected, upon account of their ſmallneſs with reſpect to the other Terms, ſo as to leave the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} = 0$, for finding the firſt approximation of v.

By extracting the Root of this Equation, we have

$$v = \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}.$$ That is,

Firſt, $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

Sec. $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $-x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

3. $\frac{\dot x}{\ddot x} - \sqrt{\frac{\dot x^2}{\ddot x^2} - \frac{2x}{\ddot x}}$, if $x - \dot x v + \frac{\ddot x v^2}{2}$, &c. $= 0$.

4. $\frac{\dot x}{\ddot x} - \sqrt{\frac{\dot x^2}{\ddot x^2} + \frac{2x}{\ddot x}}$, if $-x - \dot x v + \frac{\ddot x v^2}{2}$, &c. $= 0$.

This approximation gives v exact to twice as many places as there are true Figures in z, and therefore trebles the number of true Figures in the Expreſſion of y by $z + v$, which may be taken for a new Value of z, for computing a ſecond v, ſeeking other Values of x, $\dot x$, $\ddot x$, &c. Tho' when z is tolerably exact (which it may be eſteem'd when it contains two or three or more true Figures in the Value of y, according to the Number of Figures the Root is propoſed to be computed to,) the Calculation may be reſtor'd without ſo much trouble, only by taking $\sqrt{\frac{\dot x^2}{\ddot x^2} \pm \frac{2x}{\ddot x} - \frac{2\dddot x}{2.3\ddot x}v^3 - \frac{2\ddddot x}{1.2.3.4\ddot x}v^4}$ &c. inſtead of $\sqrt{\frac{\dot x^2}{\ddot x^2} \pm \frac{2x}{\ddot x}}$ taking every time for v its Value laſt computed.

From the ſame Equation $x + \dot x v + \frac{\ddot x v^2}{2} + \frac{\dddot x v^3}{1.2.3} +$ &c. $= 0$, may be gather'd alſo a rational Form, *viz.* $v = \frac{-x}{\dot x - \frac{x\ddot x}{2\dot x}}$. For neglecting the Terms $\frac{\dddot x v^3}{1.2.3}$, &c. we have $v = \frac{-x}{\dot x + \frac{\ddot x}{2}v}$ which is nearly $= \frac{-x}{\dot x}$. Therefore in the Diviſor inſtead of v writing $\frac{-x}{\dot x}$ we have more exactly $v = \frac{-x}{\dot{x}-\frac{\ddot{x}x}{2\dot{x}}}$, that is

1. $\frac{-x}{\dot{x}-\frac{\ddot{x}x}{2\dot{x}}}$, when $x+\dot{x}v+\frac{\ddot{x}v^2}{2}$ &c. $=0$.

2. $\frac{x}{\dot{x}+\frac{\ddot{x}x}{2\dot{x}}}$, when $-x+\dot{x}v+\frac{\ddot{x}v^2}{2}$ &c. $=0$.

3. $\frac{x}{\dot{x}-\frac{\ddot{x}x}{2\dot{x}}}$, when $x-\dot{x}v+\frac{\ddot{x}v^2}{2}$ &c. $=0$.

4. $\frac{-x}{\dot{x}+\frac{\ddot{x}x}{2\dot{x}}}$, when $-x-\dot{x}v+\frac{\ddot{x}v^2}{2}$ &c. $=0$.

This *Formula* will alſo triplicate the number of true Figures in z. And the Calculation may be repeated, after every Operation, taking for a Diviſor $\dot{x}+\frac{\ddot{x}}{2}v+\frac{\dddot{x}v^2}{1.2.3}+\frac{\ddddot{x}v^3}{1.2.3.4}$ + &c. inſtead of $x+\frac{\ddot{x}x}{2\dot{x}}$.

Dr. *Halley* has fully explain'd the manner of uſing both theſe *Formula's* in Æquations of the common Form; wherefore I ſhall be the ſhorter in explaining two or three Examples of another ſort.

Ex. I. Let it be propoſed to find the Root of this Equation $\overline{y^2+1}|^{\sqrt{2}}+y-16=0$. In this Caſe, for y writing z, and for o writing x, we have $\overline{z^2+1}|^{\sqrt{2}}$

$+z$

$+ z - 16 = x$. Whence by taking the Fluxions, we have $\dot{x} = 2\sqrt{2} \times z \times \overline{z^2 + 1}|^{\sqrt{2}-1} + 1$, and $\ddot{x} = 2\sqrt{2} \times 8 - 4\sqrt{2}\, z^2 \times \overline{z^2 + 1}|^{\sqrt{2}-2}$. For finding the firſt Figures of the Root y, for $\sqrt{2}$ take $\frac{3}{2}$, and we have the Equation $\overline{y^2 + 1}|^{\frac{3}{2}} + y - 16 = 0$, which being expanded gives $y^6 + 3y^4 + 2y^2 + 32y - 255 = 0$.

By this Equation I find that for the firſt ſuppoſition we may take $z = 2$. Therefore in order to find v, let us now make $\sqrt{2} = \frac{7}{5}$, (which is nearer than before) and we have $x = \overline{z^2 + 1}|^{\frac{7}{5}} + z - 16 = \overline{2^2 + 1}|^{\frac{7}{5}} - 14 = 5^{\frac{7}{5}} - 14 = -4{,}48$; $\dot{x} = 10{,}66$; $\ddot{x} = 4{,}72$. Whence by the ſecond rational Form $v = \dfrac{4{,}48}{10{,}66 + \dfrac{4{,}72 \times 4{,}48}{2 \times 10{,}66}} = 0{,}38$; which muſt be too big, becauſe $\frac{7}{5} < \sqrt{2}$, and therefore will require a larger Value of y to exhauſt the Equation, than where $\sqrt{2}$ is exact. For the ſecond ſuppoſition therefore, let us take $z = 2{,}3$, and make $\sqrt{2} = 1{,}4142136$, and by help of the Logarithms we ſhall have $\overline{z^2 + 1}|^{\sqrt{2}} = 13{,}47294$, whence $x = -0{,}22706$; $\dot{x} = 14{,}93429$, and $\ddot{x} = 5{,}18419$. Hence by the 2*d*. irrational *Formula* $v = \sqrt{\dfrac{14{,}93429^2}{5{,}18419^2} + \dfrac{0{,}45412}{5{,}18419}} - \dfrac{14{,}93429}{5{,}18419} = 0{,}01516$, which gives $y = z + v = 2{,}31516$, which is true to ſix Places. If you deſire it more exact than to the extent of the Tables of Logarithms, taking $z = 2{,}31516$ for the next ſuppoſition, the Calculation muſt be repeated by computing of $\overline{zz + 1}|^{\sqrt{2}}$ to a ſufficient number of Places; which muſt be done by the Binomial Series, or by making a Loga-

rithm

rithm on purpoſe, true to as many places as are neceſſary.

Ex. II. For another Example, let it be required to find the Number whoſe Logarithm is 0, 29, ſuppoſing we had no other Table of Logarithms but Mr. *Sharps* of 200 Logarithms to a great many places. This amounts to the reſolving this Equation $ly = 0{,}\,29$, or $ly - 0{,}\,29 = 0$. Hence therefore we have $x = lx - 0{,}\,29$, $\dot{x} = \frac{a}{z}$ (a being the *Modulus* belonging to the Table we uſe, *viz.* 0, 4342944819, *&c.*) $\ddot{x} = \frac{-a}{z^2}$, $\dddot{x} = \frac{2a}{z^3}$, $\ddddot{x} = \frac{-6a}{z^4}$ *&c.* In this Caſe becauſe $\ddot{x}$ has a negative Sign, changing the Signs of all the Coefficients, the Canon for v will be found in the fourth Caſe, which in the irrational Form gives $v = \frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2.3\ddot{x}}v^3 - \frac{2\ddddot{x}}{2.3.4\ddot{x}}v^4 \text{ \&c.}} = z - \sqrt{z^2 + \frac{2lz - 0{,}\,58}{a} \times z^2 + \frac{2v^3}{3z} - \frac{2v^4}{4z^2} + \frac{2v^5}{5z^3}}$ *&c.* In this Caſe to avoid often dividing by z, it will be moſt convenient to compute $\frac{v}{z}$, which is got from this Equation $\frac{v}{z} = 1 - \sqrt{1 + \frac{2lz - 0{,}\,58}{a} + \frac{2v^3}{3z^3} - \frac{2v^4}{4z^4} + \frac{2v^5}{5z^5}}$, *&c.* The neareſt Logarithm, in the Tables propoſed, to the propoſed Logarithm 0, 29 is 0, 2900346114, its Number being 1, 95. Therefore for the firſt ſuppoſition taking $z = 1{,}\,95$, we have x ($= lz - 0{,}\,29 = 0{,}\,2900346114 - 0{,}\,29$) =

0, 0000

0,0000346114, and $\frac{2\,lz - 0,58}{a} = \frac{0,0000692228}{0,4342944819} =$ 0, 00015939139, and $1 + \frac{2\,lz - 0,58}{a} = 1,00015939139$ Whence for the firſt approximation we have $\frac{v}{z} = 1 - \sqrt{1,00015939139} = -0,00007969247$, and $v = -0,00015540032$, and $y = z + v = 1,94984459968$. Which is true to eleven places, and may eaſily be corrected by the Terms $\frac{2\,v^3}{3\,z}$ &c. which I leave to the Readers curioſity.

Being upon the Subject of Approximations, it may not be amiſs to ſet down here two Approximations I have formerly hit upon The one is a Series of Terms for expreſſing the Root of any Quadratick Equation: and the other is a particular Method of Approximating in the invention of Logarithms, which has no occaſion for any of the Tranſcendental Methods, and is expeditious enough for making the Tables without much trouble.

A general Series for expreſſing the Root of any Quadratick Equation.

Any Quadratick Equation being reduc'd to this Form $xx - mqx + my = 0$, the Root x will be expreſt by this Series of Terms.

$$x = \frac{y}{q} + A \times \frac{1}{\frac{mq^2}{y} - 2} + B \times \frac{1}{a^2 - 2} + C \times \frac{1}{b^2 - 2}$$

$+ D \times \frac{1}{c^2 - 2}$ &c. Which muſt be thus interpreted.

1. The Capital Letters A, B, C, &c. ſtand for the whole Terms with their Signs, preceding thoſe wherein

in they are found, as $B = A \times \frac{1}{\frac{mq^2}{y} - 2}$.

2. The little Letters $a, b, c,$ &c. in the Diviſors, are equal to the whole Diviſors of the Fraction in the Terms immediately preceding; thus $b = a^2 - 2$.

For an Example of this, let it be required to find $\sqrt{2}$. Putting $\sqrt{2} = x + 1$, we have $x^2 + 2x - 1 = 0$, which being compared with the general *Formula*, gives $mq = -2$, and $my = -1$: therefore for m taking -1, we have $q = 2$, and $y = 1$, which Values ſubſtituted in the Series give $x = \frac{1}{2} - \frac{1}{2 \times 6} - \frac{1}{2 \times 6 \times 34} - \frac{1}{2 \times 6 \times 34 \times 1154} - \frac{1}{2 \times 6 \times 34 \times 1154 \times 1331714}$, &c. The Fractions here wrote down giving the Root true to twenty three Places.

A new Method of computing Logarithms.

This Method is founded upon theſe Conſiderations.

1. That the Sum of the Logarithms of any two Numbers is the Logarithm of the Product of thoſe two Numbers Multiplied together.

2. That the Logarithm of Unite is nothing; and conſequently that the nearer any Number is to Unite, the nearer will its Logarithm be to 0. 3*dly*. That the Product by Multiplication of two Numbers, whereof one is bigger, and the other leſs than Unite, is nearer to Unite than that of the two Numbers which is on the ſame ſide of Unite with its ſelf; for Example the two Numbers being $\frac{2}{3}$ and $\frac{4}{3}$, the Product $\frac{8}{9}$ is leſs than Unite, but nearer to it than $\frac{2}{3}$, which is alſo leſs than Unite. Upon theſe Conſiderations, I found the preſent Approximation;

proximation; which will be beſt explain'd by an Example. Let it therefore be propoſed to find the Relation of the Logarithms of 2 and of 10. In order to this, I take two Fractions $\frac{128}{100}$ and $\frac{8}{10}$, *viz.* $\frac{\overline{2}|^{7}}{\overline{10}|^{2}}$ and $\frac{\overline{2}|^{3}}{\overline{10}|^{1}}$ whoſe Numerators are Powers of 2, and their Denominators Powers of 10; one of them being bigger, and the other leſs than 1. Having ſet theſe down in Decimal Fractions in the firſt Column of the Table annext, againſt them in the ſecond Column I ſet A and B for their Logarithms, expreſſing by an Equation the manner how they are Compounded of the Logarithms of 2 and 10, for which I write $l\,2$ and $l\,10$. Then Multiplying the two Numbers in the firſt Column together, I have a third Number 1,024, againſt which I write C for its Logarithm, expreſſing likewiſe by an Equation in what manner C is formed of the foregoing Logarithms A and B. And in the ſame manner the Calculation is continued; only obſerving this *Compendium*, that before I Multiply the two laſt Numbers already got in the Table, I conſider what Power of one of them muſt be uſed to bring the Product the neareſt to Unite that can be. This is found, after we have gone a little way in the Table, only by Dividing the Differences of the Numbers from Unite one by the other, and taking the Quotient with the neareſt, for the Index of the Power wanted. Thus the two laſt Numbers in the Table being 0, 8 and 1, 024, their Differences from Unit are 0, 200 and 0, 024; therefore $\frac{0,200}{0,024}$ gives 9 for the Index; wherefore Multiplying the ninth Power of 1, 024 by 0, 8, I have the next Number 0, 990352031429, whoſe Logarithm is $D = 9\,C + B$. In ſeeking the Index in this manner by Diviſion of the Differences, the Quotient ought generally to be taken

with

with the leaſt: but in the preſent caſe it happens to be the moſt, becauſe inſtead of the Difference between 0, 8 and 1, we ought ſtrictly to have taken the difference between the reciprocal 1, 25 and 1, which would have given the Index 10; and that would be too big, becauſe the Product by that means would have been bigger than 1, as 1, 024 is. Whereas this Approximation requires that the Numbers in the firſt Column be alternately greater and leſs than 1, as may be ſeen in the Table.

When I have in this manner continued the Calculation, till I have got the Numbers ſmall enough, I ſuppoſe the laſt Logarithm to be equal to nothing. Which gives me an Equation, from which having got away the Letters by means of the foregoing Equations, I have the relation of the Logarithms propoſed. In this manner if I ſuppoſe $G=0$, I have $2136\,l\,2-643\,l\,10=0$. Which gives the Logarithm of 2 true in ſeven Figures, and too big in the Eighth; which happens becauſe the Number corresponding with G is bigger than Unite.

There is another Expedient which renders this Calculation ſtill ſhorter. It is founded upon this Conſideration, that when x is very ſmall $\overline{1+x}|^n$ is very nearly $1+nx$. Hence if $1+x$, and $1-z$ are the two laſt Numbers already got in the firſt Column of the Table, and their Powers $\overline{1+x}|^m$ and $\overline{1-z}|^n$ are ſuch as will make the Product $\overline{1+x}|^m\times\overline{1+z}|^n$ very near to Unite, m and n may be found thus: $\overline{1+x}|^m=1+mx$, and $\overline{1-z}|^n=1-nz$, and conſequently $\overline{1+x}|^m\times\overline{1-z}|^n=1+mx-nz-mnzx$, or (neglecting $mnzx$) $1+mx-nz$. Make this equal to 1, and we have $m:n::z:x::l\,\overline{1-z}:l\,\overline{1+x}$. Whence $x\,l\,\overline{1-z}+z\,l\,\overline{1+x}=0$. To give an Example of the Application of this, let 1, 024 and 0, 990352 be the laſt Numbers in the Table, their Logarithms being C and D. Then we have

1, 024

$1,024 = 1 + x$, and $0,990352 = 1 - z$, and conſequently $x = 0,024$, and $z = 0,009648$. Whence the Ratio $\frac{z}{x}$ in the leaſt Numbers is $\frac{201}{500}$. So that for finding the Logarithms propoſed we may have $500\,D + 201\,C = 48510\,l2 - 14603\,l10 = 0$, which gives $l2 = 0,3010307$, which is too big in the laſt Figure; but it is nearer the truth, than what is got from the Logarithm F ſuppoſed equal to nothing. So that by this means we have ſaved four Multiplications, which were neceſſary to find the Number 998959&c. correſpondent to F, and which muſt have been had if we would make the Logarithm true to the ſame Number of places without this *Compendium*.

1,280000000000	$A = 7\,l2 - 2\,l10$	$l2 > 0,28$
0,800000000000	$B = 3\,l2 - l10$	$< 0,33$
1,024000000000	$C = B + A = 10\,l2 - 3\,l10$	$> 0,300$
0,990352031429	$D = 9\,C + B = 93\,l2 - 28\,l10$	$< 0,30107$
1,004336277664	$E = 2\,D + C = 196\,l2 - 59\,l10$	$> 0,301020$
0,998959536107	$F = 2\,E + D = 485\,l2 - 146\,l10$	$< 0,3010309$
1,000162894165	$G = 4\,F + E = 2136\,l2 - 643\,l10$	$> 0,30102996$
0,999936281874	$H = 6\,G + F = 13301\,l2 - 4004\,l10$	$< 0,301029997$
1,000035441215	$I = 2\,H + G = 28738\,l2 - 8651\,l10$	$> 0,3010299951$
0,999971720830	$K = I + H = 42039\,l2 - 12655\,l10$	$< 0,3010299959$
1,000007161046	$L = K + I = 70777\,l2 - 21306\,l10$	$> 0,30102999562$
0,999993203514	$M = 3\,L + K = 254370\,l2 - 76573\,l10$	$< 0,30102999567$
1,000000364511	$N = M + L = 325147\,l2 - 97879\,l10$	$> 0,301029995663 5$
0,999999764687	$O = 18\,N + M = 6107016\,l2 - 1838395\,l10$	$< 0,3010299956640$
Com. Ar. 235313		
$0 = 364511\,O + 235313\,N = 2302585825187\,l2 - 693147400972\,l10$		$> 0,301029995663987$

I have computed this Table ſo far, that the Reader may ſee in what manner this Method Approximates; this whole Work, as it appears, coſting a little more than three Hours time.

V. *Proprietates quædam ſimplices* Sectionum Conicarum *ex natura* Focorum *deductæ; cum Theoremate generali de Viribus Centripetis; quorum ope Lex Virium Centripetarum ad Focos Sectionum tendentium, Velocitates Corporum in illis revolventium, & Deſcriptio Orbium facillime determinantur. Per*

Abr. *de* Moivre. R. S. Soc.

Sit DE Axis Tranſverſus Ellipſeos, AO Axis alter, & C centrum Sectionis. Sit P punctum quodvis in circumferentia ejus; PQ Tangens curvæ ad P, occurrens Axi Tranſverſo ad Q; puncta S, F Foci; CP, CK ſemidiametri Conjugatæ; PH Semilatus rectum ad diametrum PC; PG normalis ad Tangentem, cui occurrat HG, perpendicularis ipſi PCH, in puncto G, ut fiat PG radius Curvaturæ Ellipſeos in puncto P: ſint etiam ST, CR, FV perpendiculares in Tangentem PQ demiſſæ: Jungatur SO, & demittatur in Axem normalis PL. His poſitis, Dico quod,

I. *Rectangulum ſub diſtantiis ab utroque Ellipſeos Foco, ſive $SP \times PF$ æquale eſt quadrato Semidiametri CK.*

Demonſtratio.

$PSq = PCq + CSq - 2CS \times CL$ *per* 13. II. Elem.

$PFq = PCq + CSq + 2CS \times CL$ *per* 12. II. Elem.

Unde $PSq + PFq = 2PCq + 2CSq$.

Jam $PS + PF = DE = 2CD$; ac propterea

$PSq + PFq + 2PS \times PF = 4CDq$. Quare